The Law of Exponential Change - Growth and Decay

If you are a budding environmental scientist, archeologist. physical scientist, or bacteriologist, then this is the section for you. This section is where we will be looking at the **differential equation of proportional change and how it is related to the laws of decay and growth**.

**** Note:** I expect you to understand the process of developing the law of proportional growth and decay which I am about to go through, but you will not have to go through this process each time you do a problem. If you recognize the "set-up", you can go straight to the equation that is called the **Law of Exponential Change**. I suggest you memorize it.

Let's start with the differential equation of proportional change and see how we get the basic equation of growth and decay:

$\frac{dy}{dt} = k\,y$	This tells us that the rate of change of y (its derivative with respect to time) is proportional by some constant k to the amount of y present at any time.
$\frac{1}{y}\frac{dy}{dt} = k$ $\frac{1}{y}dy = k\,dt$	I moved the y to the left side and then wrote the equation in terms of differentials.
ln y = k t + C	Now integrate both sides of the equation.
$y = e^{kt+C}$ $y = e^{kt} * e^{C}$	Now let's change to exponential form. Rewrite by separating the factors.
$y = Ae^{kt}$	Since e^{C} is a constant, I can just write A instead. This is the basic equation for exponential growth or decay. **When k is positive we have growth. If k is negative we are dealing with decay.**

However, we often don't see the equation in this form; we can solve for A.

$y_0 = Ae^{k*0}$	I sub in the values at t = 0.
$y_0 = Ae^{0} = A*1$ $y_0 = A$	Simplify to find C.
$y = y_0 e^{kt}$	Therefore our final equation of growth and decay is $y = y_0 e^{kt}$ where y_0 is the amount of material initially present at time 0. This is called the **Law of Exponential Change.**

It probably makes sense to you that the **k in our equation** will be some negative constant when we are dealing with the decay of a substance and a positive constant when we have the growth of a substance. Each material or substance will have its own **rate constant, k**. Always remember that the **y stands for the amount that is present at time t**, not the part which is gone!

As I told you at the beginning of this lesson, **you do not have to start from scratch each time you do one of these problems.** If you have a problem which gives you the differential equation of $\frac{dV}{dt} = \left(-\frac{1}{40}\right)V$, you can see that it is in the same form as our original problem dy/dt = ky, so you know that the Growth and Decay formula will be the same as the one above, with k = -1/40. In this example, I can start with the equation $V = V_0 e^{(-1/40)t}$. You will still probably have to find V_0, using the info you are given in initial conditions but if you are told what your initial amount is, then that is done for you too.

So, bottom line, memorize the relationship between the proportional equation of change and the exponential law of change (also known as the growth and decay formula).
When you see the top proportion, you can directly go to the bottom formula.

Differential Equation of Proportional Change
$$\frac{dy}{dt} = k\,t$$ Where: y is the amount of material present at time t k is the growth constant .
Law of Exponential Change
$$y = y_0 e^{kt}$$ Where: y is the amount of material present at time t, y_0 is the amount of material originally present, k is the growth constant